Exponential and Logarithmic Equations

Key Points:

- For any algebraic expressions S and T and any positive real number b, $b^S = b^T$ if and only if $S = T$.
- For any algebraic expression S and positive real numbers b and c, where $b \neq 1$, $\log_b(S) = c$ if and only if $b^c = S$.
- For any algebraic expressions S and T and any positive real number b, where $b \neq 1$, $\log_b(S) = \log_b(T)$ if and only if $S = T$.

Exponential and Logarithmic Equations Video

- Solving an Exponential Equation with a Common Base: Example 1
- Solving Equations by Rewriting to have a Common Base: Examples 2-3
- Solving Equation with Positive and Negative Power: Example 4
- Solving an Equation containing Powers of Different Bases: Example 5
- Solving an Exponential Equation of the form y=Ae(kt): Example 6
- Solving an equation that can be simplified to the form y=Ae^(kt): Example 7
- Solving Exponential Equations in Quadratic Form: Example 8
- Using Algebra to Solve Logarithmic Equations: Examples 9-11
- Solving an Equation using the One to One Property of Logarithms: Examples 12-14

Practice Exercises

Follow the directions for each exercise below:

1. Solve $\mathbf{216^{3x} * 216^{x} = 36^{3x+2}}$ by rewriting each side with a common base.

2. Use logarithms to find the exact solution for $\mathbf{7 * 17^{-9x} - 7 = 49.}$ If there is no solution, write *no solution.*

3. Use logarithms to find the exact solution for $\mathbf{3e^{6n-2} + 1 = -60.}$ If there is no solution, write *no solution.*

4. Find the exact solution for $\mathbf{5e^{3x} - 4 = 6.}$ If there is no solution, write *no solution.*

5. Find the exact solution for $\mathbf{2e^{5x-2} - 9 = -56.}$ If there is no solution, write *no solution.*

6. Find the exact solution for $\mathbf{5^{2x-3} = 7^{x+1}.}$ If there is no solution, write *no solution.*

7. Find the exact solution for $\mathbf{e^{2x} - e^{x} - 110 = 0.}$ If there is no solution, write *no solution.*

8. Use the definition of a logarithm to solve: $\mathbf{-5\log_7(10n) = 5.}$

9. Use the definition of a logarithm to find the exact solution for $\mathbf{9 + 6\ln(a + 3) = 33.}$

10. Use the one-to-one property of logarithms to find an exact solution for

$\log_8(7) + \log_8(-4x) = \log_8(5)$. If there is no solution, write *no solution.*

11. Use the one-to-one property of logarithms to find an exact solution for $\ln(5) + \ln(5x^2 - 5) = \ln(56)$. If there is no solution, write *no solution.*

12. Rewrite $16^{3x-5} = 1000$ as a logarithm. Then apply the change of base formula to solve for x using the natural log. Round to the nearest thousandth.

13. Use logarithms to find the exact solution for $-9e^{10a-8} - 5 = -41$. If there is no solution, write *no solution.*

14. Find the exact solution for $10e^{4x+2} + 5 = 56$. If there is no solution, write *no solution.*

15. Find the exact solution for $-5e^{-4x-1} - 4 = 64$. If there is no solution, write *no solution.*

16. Find the exact solution for $2^{x-3} = 6^{2x-1}$. If there is no solution, write *no solution.*

17. Find the exact solution for $e^{2x} - e^x - 72 = 0$. If there is no solution, write *no solution.*

18. Use the definition of a logarithm to find the exact solution for $4\log(2n) - 7 = -11$.

19. Use the one-to-one property of logarithms to find an exact solution for $\log(4x^2 - 10) + \log(3) = \log(51)$. If there is no solution, write *no solution.*

Answers:

1. $x = \frac{2}{3}$

2. $x = \frac{\log_{\frac{1}{17}}(8)}{9}$

3. No solution.

4. $x = \frac{\ln(2)}{3}$

5. No solution.

6. $x = \log_{\frac{25}{7}}(875)$

7. $x = \ln(11)$

8. $n = \frac{1}{70}$

9. $a = e^4 - 3$

10. $x = -\frac{5}{28}$

11. $x = \pm\frac{9}{5}$

12. $x \approx 2.497$

13. $a = \frac{\ln(4)+8}{10}$

14. $x = \frac{\ln(\frac{51}{10})-2}{4}$

15. No solution.

16. $x = \log_{18}(\frac{3}{4})$

17. $x = \ln(9)$

18. No solution

19. $x = \pm\frac{3\sqrt{3}}{2}$